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### Light Scattering of PDLCS Without a Voltage for Non-Collimated System

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## LIGHT SCATTERING OF PDLCS WITHOUT A VOLTAGE FOR NON-COLLIMATED SYSTEM

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**Abstract** We have calculated the scattering properties of a single droplet with uniform and radial structures in PDLCS in the absence of a voltage for a non-collimated system such as a real PDLc projector. In these calculations, we have developed Zumer's result for a collimated system based on the anomalous diffraction approach. The scattering properties of PDLcs depend not only on droplet properties, e.g. refractive indices, radius and director configuration, but also on the film thickness and the number density of droplets. Assuming a constant concentration, we have shown that the scattering properties of PDLcs can be discussed in terms of an efficiency factor, which removes the dependence on thickness and number density. The differences between collimated and non-collimated systems are discussed as well as the effects of refractive indices, droplet radius and angular dependence of the scattering.

## INTRODUCTION

Polymer dispersed liquid crystals (PDLcs)<sup>1-4</sup> are suitable for realising high brightness displays because they do not need any polarizers, unlike conventional liquid crystal displays, e.g. twisted nematic (TN) and super twisted nematic (STN). PDLcs consist of dispersed droplets containing liquid crystals in a surrounding polymer. By applying a voltage, they can be switched from a scattering state to a transparent state, i.e. OFF state to ON state and

this is especially suitable for projector applications.<sup>5-7</sup> A conventional liquid crystal projector using a TN display cannot achieve high brightness as the polarizers absorb more than half of the incident light. On the other hand, a projector using PDLCs can achieve high brightness by using a proper optical system.

In an ideal PDLC projector, collimated light impinges on a PDLC panel and only unscattered light can reach the screen (collimated system). Therefore a strongly scattering OFF state becomes so dark that we can obtain a high contrast ratio. On the other hand, in a real PDLC projector, non-collimated light impinges on the PDLC and part of the scattered light reaches the screen as well as some unscattered light (non-collimated system).<sup>5-7</sup>

In PDLCs the difference of refractive indices between liquid crystals in a droplet and a surrounding polymer is relatively small; e.g. extraordinary and ordinary refractive indices of liquid crystals,  $n_e$  and  $n_o$  are about 1.7 and 1.5 respectively and a refractive index of a surrounding polymer,  $n_m$  is normally chosen to be the same as  $n_o$ . Under this condition, there are two relatively simple approximations; the Rayleigh-Gans approximation (RGA)<sup>8-10</sup> for particles smaller than the wavelength of light and the anomalous diffraction approach (ADA)<sup>8,11,12</sup> for larger particles.

In comparison with ADA, RGA is restricted to a relatively small degree of scattering.<sup>8</sup> In order to obtain a high contrast ratio in a PDLC projector, droplets of size corresponding to the ADA region are preferable. It has been shown that a single particle model based on ADA<sup>11</sup> can qualitatively explain the scattering properties of PDLCs in the ON state.<sup>12,13</sup> However it has also been reported that there are experimental results which cannot be explained by a single particle model in the OFF state<sup>13</sup> and that multiple scattering occurs in the real PDLCs which contain many droplets<sup>14</sup>. Nevertheless the scattering properties from many droplets should reflect those from a single droplet. We

discuss the scattering properties of PDLCS in the OFF state qualitatively using a calculation for a single droplet by ADA.

The scattering properties of PDLCS depend not only on droplet properties; e.g. the refractive indices, the radius and the director configuration, but also on the thickness of PDLCS layer and the number density of droplets. We exclude the discussion of the dependence on thickness and number density by assuming a constant concentration of liquid crystal and the consideration about an applied voltage dependence.

In this paper, we calculate the scattering properties of a single droplet with uniform and radial structures for the non-collimated system. We consider the uniform structure as an approximation of the bipolar structure which is preferable with a tangential surface anchoring.<sup>15</sup> We assume that the directors in the OFF state are uniformly oriented and normal to those in the ON state. On the other hand, with normal surface anchoring, the radial structure is preferable.<sup>15</sup>

In these calculations, we use simple expressions for the scattering cross section and the differential scattering cross section of the uniform and radial structure for collimated incident light. They have been given by Zumer based on the anomalous diffraction approach.<sup>11</sup> We discuss the difference between the collimated system and the non-collimated system and the effects of the refractive indices and the droplet radius. We also discuss the scattering profile.

#### EFFICIENCY FACTOR FOR NON-COLLIMATED SYSTEM

Fig.1 shows a schematic view of a typical projector system using PDLCS. Light emitted from the lamp is reflected by the elliptical mirror, it then goes through the aperture 1, lens 1, PDLCS panel, lens 2, aperture 2 and lens 3 and reaches the screen. The PDLCS panel consists of many pixels in which proper voltages are applied to the PDLCS independently of

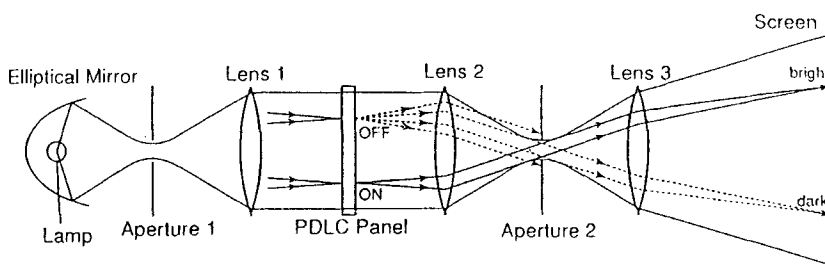


FIGURE 1 Schematic view of a typical PDLC projector

each other. We can make an image of scattering-transparent pattern on the PDLC panel. This image is projected on to the screen.

The elliptical mirror converges light at the aperture 1. Lens 1 changes this light into an almost collimated beam as aperture 1 is in the focal plane of lens 1. Lens 2 converges the collimated light at aperture 2 which is in the focal plane of lens 2. In the following, for simplicity we consider the case when the focal lengths of both lens 1 and 2 are the same.

Ideally, the light which impinges the PDLC panel should be perfectly collimated light. In this case, while the PDLC is transparent, the light would converge to a point on the aperture 2. Setting a pin-hole for aperture 2, we can achieve very high contrast ratio because scattered light is blocked in the OFF state. In order to make the collimated light by lens 1, aperture 1 should also be a pin-hole. However, the light intensity transmitted by the pin-hole at aperture 2, i.e. the light reaching the screen, becomes very small, because the light source of the lamp has a finite size and the elliptical mirror cannot converge the light to a point.

Generally, we set aperture 1 to be of a finite size which gives enough screen brightness and aperture 2 the same size as aperture 1. This arrangement gives the maximum contrast ratio without reducing the screen brightness. In

this case, part of the scattered light reaches the screen as well as unscattered light in the OFF state so that the contrast ratio decreases. The amount of scattered light reaching the screen depends on the focal length and the aperture size; generally, the light diffusing within less than 5 reaches the screen. We have to consider the non-collimated incident light instead of the collimated incident light and the scattered output light as well as the unscattered output light in the non-collimated system unlike in the collimated system.

When the intensity of the collimated incident light  $I_{in}$  decreases by scattering to  $I_{out}$ , the collimated light going in the same direction as  $I_{in}$ , the total scattering cross section  $\sigma$  is defined by,<sup>8</sup>

$$\sigma = 1 - I_{out} / I_{in}. \quad (1)$$

In the same way, we define the effective scattering cross section,  $\sigma_N$ , for the non-collimated incident light,

$$\sigma_N = 1 - F_{out} / F_{in}. \quad (2)$$

Here  $F_{in}$  is the total intensity of non-collimated incident light within a defined cone  $\Omega_0$ .  $F_N$  is the total intensity of unscattered output light within  $\Omega_0$ :

$$F_{in} = \int_{\Omega_0} I_{in}(\Omega) d\Omega, \quad (3)$$

$$F_N = \int_{\Omega_0} I_{out}(\Omega) d\Omega. \quad (4)$$

In the direction  $\Omega$ , the light intensity  $I_{in}(\Omega)$  decreases to  $I_{out}(\Omega)$  by scattering.

Then we define another effective scattering cross section for scattered output light,

$$\sigma_s = F_s / F_{in}, \quad (5)$$

where,

$$F_s = \int_{\Omega_0} \int_{\Omega_0} I_s(\Omega, \Omega') d\Omega d\Omega'. \quad (6)$$

Here  $I_s(\Omega, \Omega')$  is the scattered light intensity in the  $\Omega'$  direction which results from the incident light  $I_{in}(\Omega)$  in the  $\Omega$  direction.  $F_s$  is the total scattered light

intensity within  $\Omega_0$ . Each scattered light element is caused by a light component of the non-collimated incident light within  $\Omega_0$ .

Using these definitions, we can write the output light intensity  $F_{out}$  as,

$$F_{out} = F_N + F_S = F_{in}(1 - \sigma_N + \sigma_S) = F_{in}(1 - \sigma_{eff}), \quad (7a)$$

where

$$\sigma_{eff} = \sigma_N - \sigma_S. \quad (7b)$$

When the voltage  $V$  is applied to the PDLC, the free energy density of liquid crystal inside a droplet  $f$  using the one elastic constant approximation is given by the following equation;<sup>4</sup>

$$f = \frac{1}{2}K\{(\nabla \cdot \mathbf{n})^2 + (\nabla \times \mathbf{n})^2\} - \frac{1}{2}\Delta\epsilon\epsilon_0(\mathbf{E} \cdot \mathbf{n})^2, \quad (8)$$

where the unit vector  $\mathbf{n}$  represents the director of liquid crystals,  $K$  is the elastic constant,  $\Delta\epsilon$  is the anisotropy of the liquid crystalline dielectric constant,  $\epsilon_0$  is the dielectric constant of the air and  $\mathbf{E}$  is the electric field. We assume the electric field  $\mathbf{E}$  is constant in the PDLC and the strength  $E$  is given by

$$E = V/d. \quad (9)$$

Normalising by  $K/R^2$ , we obtain the dimensionless free energy density:

$$\begin{aligned} f/(K/R^2) \\ = \frac{1}{2}\{(\hat{\nabla} \cdot \mathbf{n})^2 + (\hat{\nabla} \times \mathbf{n})^2\} - \frac{1}{2}(\Delta\epsilon\epsilon_0/K)(R/d)^2 V^2 \cos^2 \psi, \end{aligned} \quad (10)$$

where  $\hat{\nabla} = R \nabla$  denotes the differential operator in the space normalised by the droplet radius  $R$ , and  $\psi$  denotes the angle between the director and the electric field.

This equation shows that the applied voltage dependence of the director configuration in a droplet is identical if  $R/d$  are the same.

Now we consider the scattering properties. The scattering properties should depend on the product  $\sigma Nd$ ,<sup>8</sup>

where  $\sigma$  is the scattering cross section,  $N$  is the number density of droplets and  $d$  is the thickness of PDLCS.  $N$  is proportional to  $R^{-3}$  for the constant concentration of liquid crystal in PDLCS and so introducing the efficiency factor,  $Q = \sigma / \pi R^2$ ,<sup>8</sup>

$$\sigma N d \propto (Q \pi R^2) (1/R^3) d = Q \pi (d/R). \quad (11)$$

Considering Eq.(10), this equation shows that the scattering properties depend only on  $Q$  if PDLCS have the same applied voltage dependence. Therefore in the following discussion, we compare the efficiency factors. We put

$$Q = \sigma / \pi R^2, \quad (12a)$$

$$Q_N = \sigma_N / \pi R^2, \quad (12b)$$

$$Q_S = \sigma_S / \pi R^2. \quad (12c)$$

$$Q_{\text{eff}} = \sigma_{\text{eff}} / \pi R^2 = Q_N - Q_S \quad (12d)$$

where,  $Q$  is the efficiency factor corresponds for the total scattered light in the collimated system,  $Q_N$  the factor for the total scattered light in the non-collimated system,  $Q_S$  is the amount of scattered light which reaches the screen and  $Q_{\text{eff}}$  the amount of scattered light which does not reach the screen.

#### ANOMALOUS DIFFRACTION APPROXIMATION

We assume that the droplet is spherical and that the director configurations in the OFF state are either the uniform structure with the directors aligned perpendicular to the direction of the applied field i.e. in the  $x$ - $y$  plane (Fig.2), or the radial structure. Also we assume that the ordinary refractive index of liquid crystal,  $n_o$ , is equal to that of the surrounding polymer,  $n_m$  ( $n_o = n_m$ ). This condition leads to no scattering for the incident light along the electric field  $\mathbf{E}$ , i.e. the  $z$ -axis in Fig.2, when sufficient voltage is applied to the PDLCS.

According to Zumer, with above assumptions, the scattering cross section of the uniform structure is given by,<sup>11</sup>



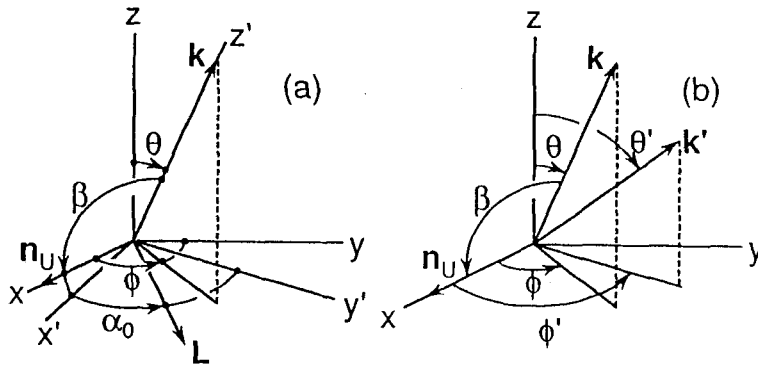


FIGURE 2 Schematic presentation of the scattering geometry for a single droplet: (a) the wave vector  $\mathbf{k}$  specified with  $\theta$  and  $\phi$ , the director of the uniform structure  $\mathbf{n}_U$  along x-axis, the polarized direction of the incident light  $\mathbf{L}$  specified with  $\alpha_0$ . (b) the incident wave vector  $\mathbf{k}$  specified with  $\theta$  and  $\phi$ , the scattered wave vector  $\mathbf{k}'$  specified with  $\theta'$  and  $\phi'$ .  $\beta$  is the angle between  $\mathbf{k}$  and  $\mathbf{n}_U$ .  $\delta$  is the angle between  $\mathbf{k}$  and  $\mathbf{k}'$ .

$$\sigma = 2\sigma_0 \cos^2 \alpha_0 H'(v_e, 0) \quad (13a)$$

where  $\sigma_0 = \pi R^2$  and  $\alpha_0$  denotes the polarized direction of the incident linear polarized light and the directors (Fig.2a).  $H'$  and  $n_e$  are given by,<sup>11</sup>

$$H'(v_e, 0) = 1 - (2/v_e) \sin v_e + (2/v_e^2) (1 - \cos v_e), \quad (14)$$

$$v_e(\beta) = 2k_0 R \{n_{\text{eff}}(\beta) - n_m\} = 2k_0 R \{n_{\text{eff}}(\beta) - n_0\}, \quad (15)$$

where

$$n_{\text{eff}}(\beta) = \{(\cos \beta / n_0)^2 + (\sin \beta / n_e)^2\}^{-1/2} \quad (16)$$

$$\beta = \cos^{-1} \{\sin \theta \cos \phi\}. \quad (17)$$

Here  $k_0 = 2\pi/\lambda$  is the wave number in the air,  $\lambda$  is the wavelength in the air,  $n_e$  is the extraordinary refractive index of liquid crystals,  $\beta$  is the angle between the wave vector  $\mathbf{k}$  and the director  $\mathbf{n}$ ,  $\theta$  and  $\phi$  define  $\mathbf{k}$  (Fig.2a).

If the incident light is perfectly non-polarized light, it is represented by the combination of an arbitrary pair of orthogonal linear polarized light waves with equal

intensity. Taking one of them along the  $x'$ -axis ( $\alpha_0=0$ ), the other along  $y'$ -axis ( $\alpha_0=\pi/2$ ) is not scattered by a droplet with the uniform structure, because it becomes perpendicular to the directors and we have assumed  $n_o=n_m$  (Fig.2a). Therefore, using Eq.(13a) the scattering cross section for the non-polarized incident light is given by

$$\sigma(\mathbf{k}) = \sigma_0 H'(\nu_e, 0). \quad (13b)$$

In the same way, using Zumer's expressions<sup>11</sup> the differential scattering cross section of the uniform structure for the non-polarized incident light is given by,

$$\sigma'(\mathbf{k}, \mathbf{k}') = \frac{(k_0 R n_m)^2}{8\pi} \sigma_0 |H(i\nu_e, k_0 R n_m \sin \delta)|^2, \quad (18)$$

where

$$H(i\nu_e, z) = 2 \int_0^1 \{1 - \exp(i\nu_e \sqrt{1-x^2})\} J_0(xz) dx, \quad (19)$$

and

$$\delta = \cos^{-1}\{\sin\theta \sin\theta' \cos(\phi - \phi') + \cos\theta \cos\theta'\}, \quad (20)$$

and  $\theta'$  and  $\phi'$  define the wave vector of the scattered light  $\mathbf{k}'$ ,  $\delta$  is the angle between the wave vectors of the incident light  $\mathbf{k}$  and the scattered light  $\mathbf{k}'$  (Fig.2b).  $J_0$  is the Bessel function of the zeroth order.

In the same way, by using Zumer's results,<sup>11</sup> we can obtain the scattering cross section and differential scattering cross section of the radial structure for the non-polarized incident light as follows;

$$\sigma(\mathbf{k}) = 2\sigma_0 \int_0^1 (1 - \cos \Delta_s) y dy \quad (21)$$

$$\sigma'(\mathbf{k}, \mathbf{k}') = \frac{(k_0 R n_m)^2}{8\pi} \sigma_0 (C_+^2 + C_-^2 + D_+^2 + D_-^2) \quad (22)$$

with,

$$\Delta_s = 2k_0 R n_m \int_0^{\sqrt{1-y^2}} \left\{ \sqrt{\frac{x^2 + y^2}{(n_m/n_s)^2 y^2 + x^2}} - 1 \right\} dx \quad (23)$$

$$C_{\pm} = \int_0^1 \{J_0(k_0 R n_m y \sin \delta) \mp J_2(k_0 R n_m y \sin \delta)\} (\cos \Delta_s - 1) y dy \quad (24a)$$

$$D_{\pm} = \int_0^1 \{J_0(k_0 R n_m y \sin \delta) \mp J_2(k_0 R n_m y \sin \delta)\} \sin \Delta_s y dy \quad (24b)$$

Taking the cone to be symmetric about the  $z$ -axis and assuming the intensity of the incident light  $I_{in}(\theta, \phi)$  is constant regardless of the angle  $\theta$  and  $\phi$ ; i.e.  $I_{in}(\theta, \phi) = I_0$ ,  $F_{in}$ ,  $F_N$  and  $F_S$  within the cone defined by the angle  $\theta_0$  are given by,

$$F_{in} = \int_0^{\theta_0} \int_0^{2\pi} I_0 d\phi \sin\theta d\theta = 2\pi(1 - \cos\theta_0) I_0, \quad (25a)$$

$$F_N = \int_0^{\theta_0} \int_0^{2\pi} (1 - \sigma(\mathbf{k})) I_0 d\phi \sin\theta d\theta, \quad (25b)$$

$$F_S = \int_0^{\theta_0} \int_0^{2\pi} \left\{ \int_0^{\theta_0} \int_0^{2\pi} \sigma'(\mathbf{k}, \mathbf{k}') I_0 d\phi \sin\theta d\theta \right\} d\phi' \sin\theta' d\theta'. \quad (25c)$$

Substituting Eqs. (13b), (18) into (25), then using Eqs. (12a)-(12c) with Eqs. (2) and (5), we obtain  $Q$ ,  $Q_N$  and  $Q_S$  of the uniform structure;

$$Q = H'(\mathbf{v}_e, 0) \quad \text{with } \theta=0, \quad (26a)$$

$$Q_N = \frac{1}{2\pi(1 - \cos\theta_0)} \int_0^{\theta_0} \int_0^{2\pi} H'(\mathbf{v}_e, 0) d\phi \sin\theta d\theta, \quad (26b)$$

$$Q_S = \frac{1}{2\pi(1 - \cos\theta_0)} \frac{(k_0 R n_m)^2}{8\pi} \times \int_0^{\theta_0} \int_0^{2\pi} \int_0^{\theta_0} \int_0^{2\pi} |H(i\mathbf{v}_e, k_0 R n_m \sin\delta)|^2 \sin\theta \sin\theta' d\phi d\theta d\phi' d\theta'. \quad (26c)$$

In the same way, substituting Eqs. (21), (22) into (25), then using Eqs. (12a), (12b) and (12c) with Eqs. (2) and (5), we obtain  $Q$ ,  $Q_N$  and  $Q_S$  of the radial structure;

$$Q = Q_N = 2 \int_0^1 (1 - \cos\Delta) y dy \quad (27a)$$

$$Q_S = \frac{1}{2\pi(1 - \cos\theta_0)} \frac{(k_0 R n_m)^2}{8\pi} \times 4\pi \int_0^{\theta_0} \int_0^{\theta_0} \int_0^{2\pi} (C_+^2 + C_-^2 + D_+^2 + D_-^2) \sin\theta \sin\theta' d\phi'' d\theta d\theta' \quad (27b)$$

with,

$$\phi'' = \phi - \phi' \quad (28)$$

In this case, we can reduce the integral procedure due to the symmetry of the radial structure.

We calculate the integral in these equations numerically by Simpson's rule.

## RESULTS

For the calculations we have chosen the refractive indices  $n_o=n_m=1.5$ , and the wavelength  $\lambda=555\text{nm}$  to which human beings are most sensitive. We define  $v$  as the factor representing the refractive index dependence where

$$v = 2k_0R(n_e-n_o) = 2k_0R(n_e-n_m). \quad (29)$$

Fig.3 and Fig.4 show  $Q$ ,  $Q_{\text{eff}}$ ,  $Q_N$  and  $Q_S$  as a function of  $v$  for  $R=0.5, 1.0, 2.0\mu\text{m}$  and  $\theta_0=5^\circ$ .  $\theta_0$  is the angle in the surrounding polymer.  $v$  is changed by changing the extraordinary refractive index of liquid crystals  $n_e$ . In Fig.3a and 4a  $Q$ ,  $Q_{\text{eff}}$ ,  $Q_N$  and  $Q_S$  of the uniform structure are calculated by Eq.(26). In Fig.3b and 4b  $Q$ ,  $Q_{\text{eff}}$ ,  $Q_N$  and  $Q_S$  of the radial structure are calculated by Eq.(27).

For the uniform structure  $Q$  does not depend up on  $R$  while  $v$  is constant. It has the 1st maximum( $Q=1.59$ ) at  $v=4.09$  and the 1st minimum( $Q=0.77$ ) at  $v=7.63$ . Then it oscillates around  $Q=1$  and this oscillation reduces with increasing  $v$ . Finally it converges to  $Q=1$ . The 1st maximum is larger than the other maxima at larger  $v$ . This property is similar to the scattering from an isotropic droplet.<sup>8</sup> From these results,  $n_e$  should be chosen to give  $v=4.09$  regardless of  $R$  in order to achieve high contrast ratio in the collimated system.

For the radial structure  $Q$  shows a similar property as for the uniform structure. However it depends on  $R$  and the period of the oscillation is longer than the uniform structure. Although it depends on  $R$ , the 1st maximum value given at about  $v=7$  is almost the same( $Q=1.74$ ) regardless of  $R$ , and it is larger than the maximum value of the uniform structure.

The non-collimated system differs from the collimated system in two ways:

- (1) non-collimated light impinges on the PDLc panel instead of collimated light, and
- (2) a part of scattered light as well as unscattered light reaches the screen. The difference between  $Q$  and  $Q_N$

for the uniform structure arises from (1) whereas (2) causes non-zero  $Q_S$  for both the uniform and radial structures. In this way,  $Q_{eff}$  for the non-collimated system should be different from  $Q$  for the collimated system.

Fig.3 shows that  $Q_{eff}$  is less than  $Q$  and  $Q_{eff}$  decreases with increasing  $R$ . The difference between  $Q_{eff}$  and  $Q$  are dominantly attributable to difference (1) rather than difference (2), because  $Q_S$  is relatively large while the difference between  $Q$  and  $Q_N$  is not remarkable even for the uniform structure as is shown in Fig.4.

The maximum value of  $Q_{eff}$  occurs at about  $v=4$  (uniform structure) and  $v=7$  (radial structure) which is similar to the maximum value of  $Q$  in the collimated system. However the maximum value of  $Q_{eff}$  decrease with increasing  $R$  while the maximum value of  $Q$  is constant regardless of  $R$ . The degree of the decrease with respect to  $R$  in  $Q_{eff}$  of the radial structure is smaller than  $Q_{eff}$  of the uniform structure.  $R$  should be as small as possible subject to  $v=4$  (uniform structure) or  $v=7$  (radial structure) in order to achieve high contrast ratio in the non-collimated system, i.e. a real PDLC projector.

Around the 1st minimum of  $Q$ ,  $Q_{eff}$  is close to  $Q$  regardless the structures (Fig.3), in other words  $Q_S$  is very small compared to  $Q_N$  because  $Q_N$  is nearly equal to  $Q$ . As  $Q_N$  corresponds to the total scattered light, these results mean that the ratios of the scattered light within the cone defined by  $\theta_0$  to the total scattered light are small around the 1st minimum of  $Q$ . Fig.5 shows that  $Q_S/Q_N$  has the minimum at the 1st minimum of  $Q$  but does not have the maximum around the 1st maximum of  $Q$ .

These properties for  $Q_{eff}$  should reflect the profile of the differential scattering cross section. Fig.6 shows the integrals of the differential scattering cross section normalized by the scattering cross section  $\sigma_C(\theta_0)/\sigma$  as a function of  $\theta_0$  when the collimated light impinges a PDLC panel where

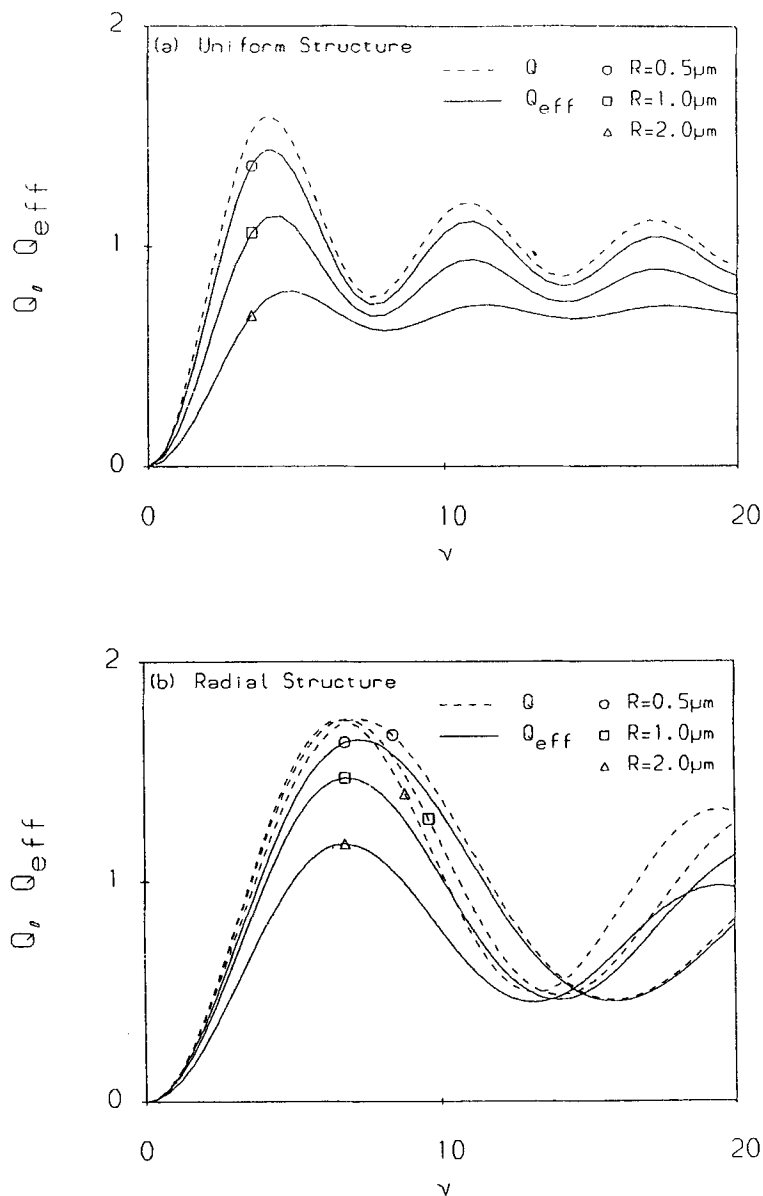


FIGURE 3 The efficiency factor in the collimated system,  $Q$ , and the non-collimated system,  $Q_{eff}$ , as a function of  $\nu=2k_0R(n_e-n_o)$  for  $R=0.5, 1.0, 2.0\mu\text{m}$  with  $\theta_0=5^\circ$ ,  $n_o=n_m=1.5$  and the wavelength  $\lambda=555\text{nm}$ : (a) uniform structure, (b) radial structure.

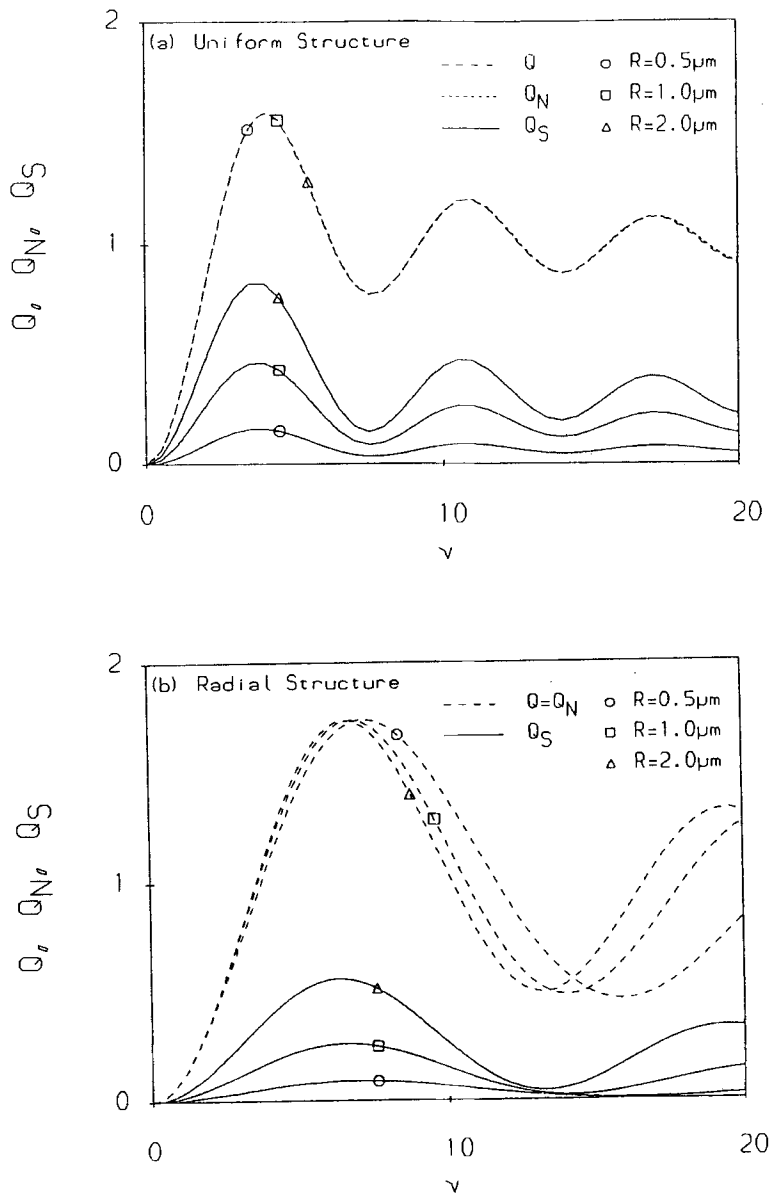


FIGURE 4 The efficiency factors in the non-collimated system,  $Q$ ,  $Q_N$  and  $Q_S$  as a function of  $\nu=2k_0R(n_e-n_o)$  for  $R=0.5, 1.0, 2.0\mu\text{m}$  with  $\theta_0=5^\circ$ ,  $n_o=n_m=1.5$  and the wavelength  $\lambda=555\text{nm}$ : (a) uniform structure, (b) radial structure.

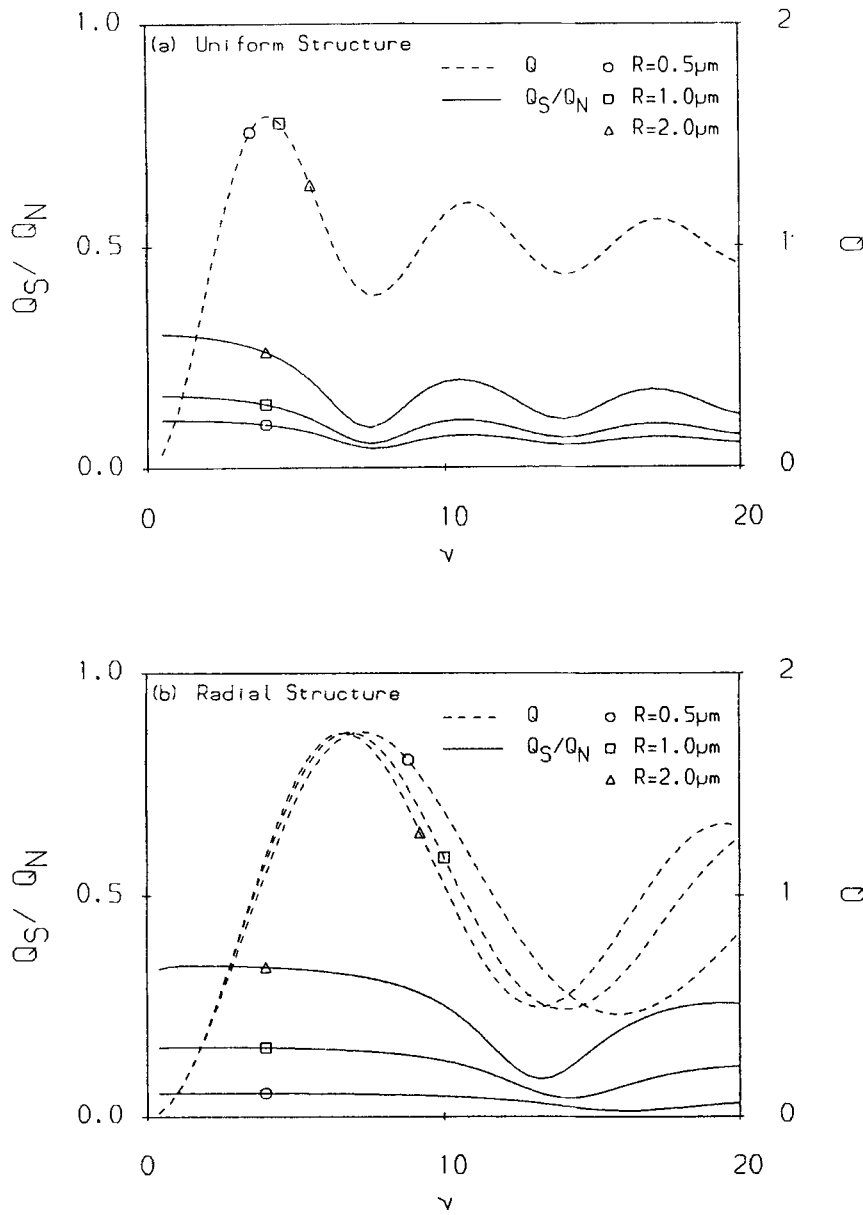


FIGURE 5 The ratio of efficiency factors,  $Q_S/Q_N$ , as a function of  $\nu=2k_0R(n_e-n_o)$  for  $R=0.5, 1.0, 2.0\mu\text{m}$  with  $\theta_0=5^\circ$ ,  $n_o=n_m=1.5$  and the wavelength  $\lambda=555\text{nm}$ : (a) uniform structure, (b) radial structure.



$$\sigma_C(\theta_0) = \int_0^{\theta_0} \int_0^{2\pi} \sigma'(\mathbf{k}, \mathbf{k}') d\phi' \sin\theta' d\theta', \quad (30)$$

and  $\mathbf{k}$  is parallel to  $z$ -axis.

Fig.6a and 6b show  $\sigma_C/\sigma$  as functions of  $\theta_0$  for the uniform structure at values of  $v$  which give the 1st maximum and the 1st minimum value of  $Q$  respectively. In Fig.6a and 6b,  $\sigma'(\mathbf{k}, \mathbf{k}')$  is given by Eq.(18) with  $\delta=\theta'$ . Fig.6c and 6d show  $\sigma_C/\sigma$  as functions of  $\theta_0$  for the radial structure at values of  $v$  which give the maximum and minimum value of  $Q$

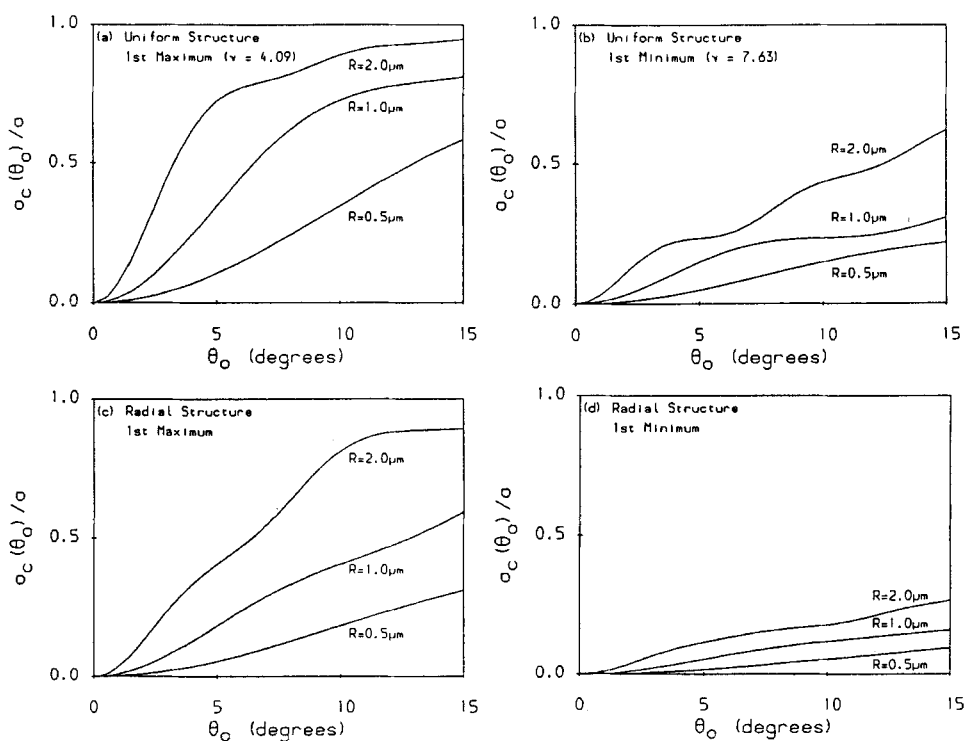


FIGURE 6 The integral of the differential scattering cross sections as functions of  $\theta_0$  for different radii  $R=0.5, 1.0, 2.0\mu\text{m}$  with  $n_0=n_m=1.5$  and the wavelength  $\lambda=555\text{nm}$ : Fig.6a and 6b show the  $\sigma_C/\sigma$  for the uniform structure at values of  $v$  which give the 1st maximum( $v=4.09$ ) and the 1st minimum( $v=7.63$ ) of  $Q$  respectively. Fig.6c and 6d show the  $\sigma_C/\sigma$  of the radial structure with  $v$  which gives the maximum and minimum value of  $Q$  respectively.

respectively. In Fig.6c and 6d,  $\sigma'(\mathbf{k}, \mathbf{k}')$  is given by Eq. (22) with  $\delta = \theta'$ . Here  $\sigma_C(\theta_0)/\sigma$  corresponds to the ratio of the amount of the scattered light inside the cone defined by  $\theta_0$  to the total scattered light when the collimated light impinges a PDLc panel.

In each figure,  $v$  is chosen to correspond to the 1st maximum or the 1st minimum of  $Q$ . Therefore the total scattered light is the same in each figure, i.e.  $\sigma$  is constant, regardless of  $R$  because the 1st maximum and the 1st minimum value of  $Q$  are invariant in the uniform structure and nearly invariant even in the radial structure. These figure shows that the scattering profile depends on  $R$  while the total scattered light is invariant. With larger  $R$ ,  $\sigma_C/\sigma$  reaches 1.0 at smaller  $\theta_0$ . This means that the scattered light is more concentrated around the direction of the incident light for large  $R$ . In other words, the scattering is "far" for small  $R$  and "near" for large  $R$  while the total scattered light is constant.

Comparing Fig.6a. and 6b or Fig.6c and 6d, we can see that the scattering is farther at the 1st minimum of  $Q$  than at the 1st maximum of  $Q$  regardless of the structures. Reconsidering the results in Fig.5, we find that the scattering is farthest at the 1st minimum of  $Q$ .

Comparing Fig.6a and 6c or Fig.6b and 6d, we can see that the scattering is farther in the radial structure than in the uniform structure with respect to the same  $R$ .

From the above discussion, we can explain the  $R$  dependence of  $Q_{\text{eff}}$  as follows: Regardless of  $R$ , the maximum value of the total scattered light is the same; i.e. the maximum value of  $Q_N$  is the same. However, the scattering becomes "farther" with decreasing  $R$ . Therefore  $Q_S$ , the light scattered within the cone defined by  $\theta_0$ , becomes small and  $Q_{\text{eff}}$  increases.

Summarising this discussion,  $R$  should be as small as possible and  $v$  should be about 4(uniform structure) or about 7(radial structure) in order to realise a high contrast ratio. Small values of  $R$  at constant  $v$  require large values

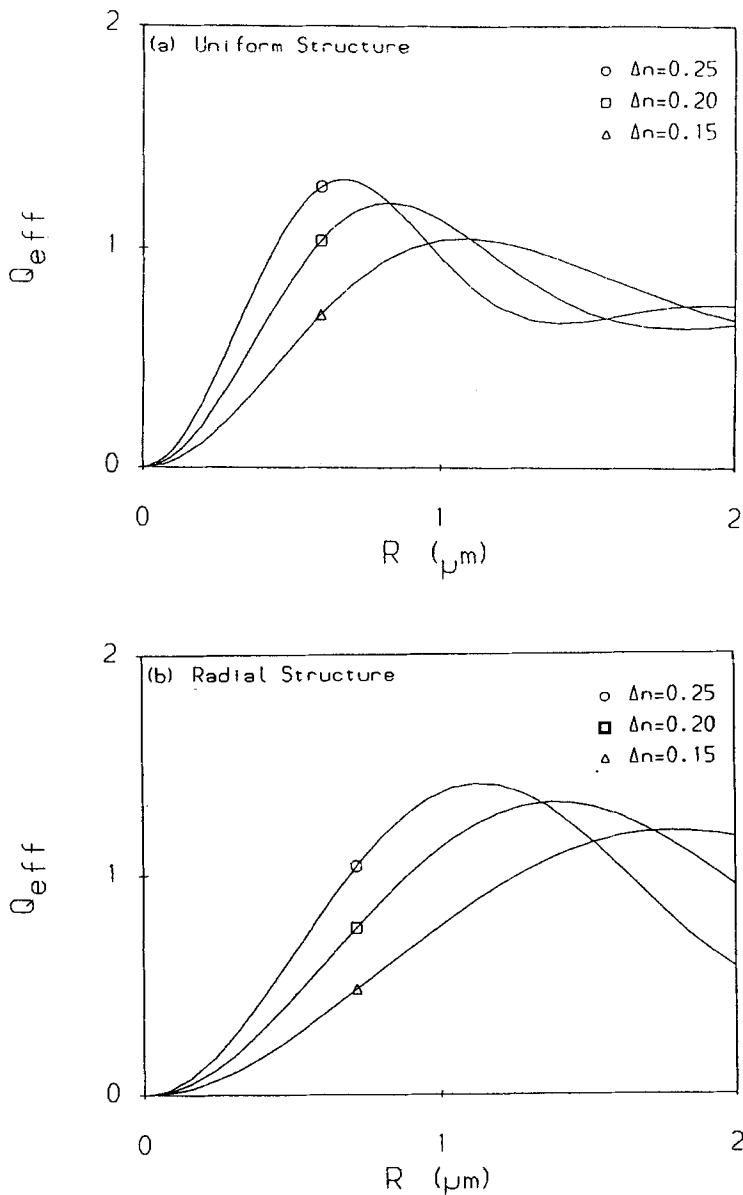


FIGURE 7 The radius dependence of the efficiency factors in the non-collimated system ( $Q_{eff}$ ) for different  $\Delta n = n_e - n_o = 0.15, 0.20, 0.25$  with  $n_o = n_m = 1.5$ , the wavelength  $\lambda = 555 \text{ nm}$ : (a) uniform structure, (b) radial structure.

of  $\Delta n = n_e - n_o$ . However generally  $\Delta n$  is restricted to be less than 0.25. This implies  $R$  should be  $0.7\mu\text{m}$  for the uniform structure and  $1.1\mu\text{m}$  for the radial structure as is shown in Fig.7.

### CONCLUSIONS

For the collimated system, the maximum value of the efficiency factor in the OFF state,  $Q$ , is the same regardless of the droplet radius  $R$ . For the uniform structure  $Q$  does not depend on  $R$  while  $v = 2k_0R(n_e - n_o)$  is constant. It becomes a maximum value 1.59 at  $v = 4.09$  regardless of  $R$ . On the other hand, for the radial structure  $Q$  slightly depends on  $R$  while  $v$  is constant. However, its maximum value is almost the same regardless of  $R$ . It is 1.74 at about  $v = 7$ . The maximum value of the radial structure is larger than the uniform structure.

On the other hand, for the non-collimated system, the efficiency factor  $Q_{\text{eff}}$  becomes a maximum at around the value of  $v$  which gives the maximum value of  $Q$ . However the maximum value of  $Q_{\text{eff}}$  is less than that of  $Q$  and it strongly depends on  $R$ . With decreasing  $R$ , the maximum value of  $Q_{\text{eff}}$  increases. Therefore, in a real projector in which non-collimated light impinges on a PDLC panel and scattered light as well as unscattered light reaches the screen, we should choose  $R$  to be as small as possible bearing in mind the restriction of  $v \leq 4$  (uniform structure) or  $v \leq 7$  (radial structure). We can realise these condition by choosing  $R = 0.7\mu\text{m}$  (uniform structure) or  $R = 1.1\mu\text{m}$  (radial structure) and  $\Delta n = n_e - n_o = 0.25$  for the light of 555nm wavelength which is the most sensitive for human beings.

The difference between  $Q_{\text{eff}}$  and  $Q$  is concerned with the two differences of the non-collimated system from the collimated system; (1) non-collimated light impinge on a PDLC panel instead of collimated light, (2) a part of the scattered light as well as the unscattered light reaches the screen. We found that the difference between  $Q_{\text{eff}}$  and  $Q$  is

mainly attributable to difference (1) rather than difference (2).

The R dependence of  $Q_{\text{eff}}$  is explained as follows: The maximum value of the intensity of the total scattered light is invariant regardless of R if we choose  $v$  properly. However the scattering profile becomes "farther" with decreasing R. This implies that the scattered light in the defined cone, i.e. that which reaches the screen, decreases while the total scattered light is invariant. Therefore the amount of the scattered light which does not reach the screen,  $Q_{\text{eff}}$ , increases.

As for the scattering profile, we found that the scattering becomes "farthest" at the value of  $v$  which gives the minimum value of the total scattered light in both uniform and radial structures.

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